Time-dependent Hartree-Fock calculations for multi-nucleon transfer and quasi-fission processes

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INTRODUCTION: Interests for multinucleon transfer reactions
INTRODUCTION: Interests for multinucleon transfer reactions

➢ Transfer reaction

Ex.) $^{40}\text{Ca} + ^{124}\text{Sn}$ reaction

$^{40}\text{Ca} + ^{124}\text{Sn} \rightarrow ^{41}\text{Ca} + ^{123}\text{Sn}$ (0p, +1n)

$^{39}\text{K} + ^{125}\text{Sb}$ (-1p, 0n)

$^{38}\text{Ar} + ^{126}\text{Te}$ (-2p, 0n)

\vdots

$^{124}\text{Sn}$

$^{40}\text{Ca}$

+ sign

- sign
Production cross sections for $^{40}$Ca-like fragments

- Horizontal axis: Number of neutrons in smaller fragment (incident $^{40}$Ca)
- Labels “($-x$p)”, $x=0, \ldots, 6$: Number of removed protons from $^{40}$Ca

Objective experimental data: $^{40}$Ca$_{20} + ^{124}$Sn$_{124}$ at $E_{\text{lab}} = 170$ MeV

Objective experimental data: \[ ^{40}\text{Ca}_{20} + ^{124}\text{Sn}_{124} \text{ at } E_{\text{lab}} = 170 \text{ MeV} \]

Production cross sections for \(^{40}\text{Ca}\)-like fragments

- Horizontal axis: Number of neutrons in smaller fragment (incident \(^{40}\text{Ca}\))
- Labels “(-xp)”, \(x=0, ..., 6\): Number of removed protons from \(^{40}\text{Ca}\)


Transfer of 5 neutrons \(^{40}\text{Ca} \rightarrow ^{124}\text{Sn}\)
Production cross sections for $^{40}\text{Ca}$-like fragments

- Horizontal axis: Number of neutrons in smaller fragment (incident $^{40}\text{Ca}$)
- Labels “(-xp)”, $x=0, \ldots, 6$: Number of removed protons from $^{40}\text{Ca}$

Objective experimental data: $^{40}\text{Ca}_{20} + ^{124}\text{Sn}_{50}$ at $E_{\text{lab}} = 170$ MeV


Transfer of 5 neutrons $^{40}\text{Ca}\leftrightarrow^{124}\text{Sn}$

Transfer of 6 protons $^{40}\text{Ca}\rightarrow^{124}\text{Sn}$
INTRODUCTION: Interests for multinucleon transfer reactions

Transfer reaction

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$^{40}\text{Ca} + ^{124}\text{Sn} \rightarrow ^{41}\text{Ca} + ^{123}\text{Sn}$ (0p, +1n)

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$\vdots$

$^{124}\text{Sn}$

$^{40}\text{Ca}$
INTRODUCTION: Interests for multinucleon transfer reactions

- Transfer reaction

Ex.) $^{40}\text{Ca} + ^{124}\text{Sn}$ reaction

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... $^{124}\text{Sn}$

- Two types of transfer mechanisms

Quantum tunneling

Neck formation & breaking
INTRODUCTION: Interests for multinucleon transfer reactions

- Transfer reaction
  
  Ex.) $^{40}\text{Ca} + ^{124}\text{Sn} \rightarrow ^{41}\text{Ca} + ^{123}\text{Sn}$ (0$p$, +1$n$)
  $^{39}\text{K} + ^{125}\text{Sb}$ (-1$p$, 0$n$)
  $^{38}\text{Ar} + ^{126}\text{Te}$ (-2$p$, 0$n$)
  ...

- Two types of transfer mechanisms

  - Quantum tunneling
  
    ![Quantum tunneling diagram]

  - Neck formation & breaking
    
    ![Neck formation & breaking diagram]

- A new means to produce neutron-rich unstable nuclei
INTRODUCTION: Interests for multinucleon transfer reactions

Figure was taken from Rept. Prog. Phys. 70, 1525 (2007)

➢ Production of $N$-rich unstable nuclei which are important to understand heavy-element synthesis in the $r$-process
INTRODUCTION: Interests for multinucleon transfer reactions


- Production of N-rich (super)heavy nuclei through MNT reactions in $^{238}_{92}$U$^{+}_{146} + ^{248}_{96}$Cm$_{152}$

K. Sekizawa
Time-dependent Hartree-Fock calculations for MNT and QF processes
INTRODUCTION: Interests for multinucleon transfer reactions

➢ Transfer reaction

Ex.) $^{40}\text{Ca} + ^{124}\text{Sn}$ reaction

$^{40}\text{Ca} + ^{124}\text{Sn} \rightarrow ^{41}\text{Ca} + ^{123}\text{Sn}$  (0p, +1n)

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$^{38}\text{Ar} + ^{126}\text{Te}$  (-2p, 0n)

➢ Two types of transfer mechanisms

Quantum tunneling

Neck formation & breaking

➢ A new means to produce neutron-rich unstable nuclei
Outline

1. Introduction

2. TDHF calculation for multinucleon transfer (MNT) processes

3. TDHF calculation for MNT & QF processes in reactions involving $^{238}\text{U}$

4. Summary and Perspective
1. Introduction

2. TDHF calculation for multinucleon transfer (MNT) processes
   a) Transfer dynamics described by the TDHF theory
   b) How to calculate transfer probabilities from a TDHF wave function
   c) Systematics of MNT processes: $N/Z$ and $Z_pZ_T$ dependence

3. TDHF calculation for MNT & QF processes in reactions involving $^{238}\text{U}$

4. Summary and Perspective
\( ^{40}\text{Ca} + ^{124}\text{Sn} \) reaction

\[ \frac{N}{Z} = 1.00 \left( ^{40}\text{Ca} \right), 1.48 \left( ^{124}\text{Sn} \right) ; \ Z_pZ_T = 1000 \]
Results of the TDHF calculation: \( ^{40}_{20}\text{Ca} + ^{124}_{50}\text{Sn} \) at \( E_{\text{lab}} = 170 \text{ MeV} \)


Density evolution obtained from the TDHF calculation

3D-grid: 60×60×26 (48 fm×48 fm×20.8 fm), Mesh size: 0.8 fm
Skyrme force: SLy5, \( \Delta t \): 0.2 fm/c, Initial separation distance: 16 fm
Calculated impact parameter: \( 0 \leq b \leq 10 \text{ fm} \)
Fusion reactions (\( b \leq 3.95 \text{ fm} \)), Binary reactions (\( b \geq 3.96 \text{ fm} \))

Projectile: \(^{40}\text{Ca}\)
Target: \(^{124}\text{Sn}\)

\( b = 3.95 \text{ fm} \)
\( b = 3.96 \text{ fm} \)
\( b = 4.50 \text{ fm} \)
Results of the TDHF calculation: $^{40}\text{Ca}_{20}+^{124}\text{Sn}_{50}$ at $E_{\text{lab}}=170$ MeV


3D-grid: $60 \times 60 \times 26$ (48 fm × 48 fm × 20.8 fm), Mesh size: 0.8 fm
Skyrme force: SLy5, $\Delta t$: 0.2 fm/c, Initial separation distance: 16 fm
Calculated impact parameter: $0 \leq b \leq 10$ fm
Fusion reactions ($b \leq 3.95$ fm), Binary reactions ($b \geq 3.96$ fm)

Density evolution obtained from the TDHF calculation

$\ b=3.95$ fm

$\ b=3.96$ fm

$\ b=4.50$ fm
Results of the TDHF calculation: $^{40}\text{Ca}_{20} + ^{124}\text{Sn}_{50}$ at $E_{\text{lab}} = 170$ MeV


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Fusion reactions ($b \leq 3.95$ fm), Binary reactions ($b \geq 3.96$ fm)

Density evolution obtained from the TDHF calculation

$h=3.95$ fm

$h=3.96$ fm

$h=4.50$ fm
Density evolution obtained from the TDHF calculation

\[ b = 3.95 \text{ fm} \quad \text{3D-grid: } 60 \times 60 \times 26 \text{ (48 fm} \times 48 \text{ fm} \times 20.8 \text{ fm)}, \text{ Mesh size: } 0.8 \text{ fm} \]

Skyrme force: SLy5, \( \Delta t \): 0.2 fm/c, Initial separation distance: 16 fm

Calculated impact parameter: \( 0 \leq b \leq 10 \text{ fm} \)

Fusion reactions \( (b \leq 3.95 \text{ fm}) \), Binary reactions \( (b \geq 3.96 \text{ fm}) \)

Results of the TDHF calculation: \( ^{40}_{20}\text{Ca} + ^{124}_{50}\text{Sn} \) at \( E_{\text{lab}} = 170 \text{ MeV} \)

Density evolution obtained from the TDHF calculation

\[ b = 3.95 \text{ fm} \]

- Fusion (\( b \leq 3.95 \text{ fm} \))

\[ b = 3.96 \text{ fm} \]

- Transfer (3.96 fm \( \leq b \))

\[ b = 4.50 \text{ fm} \]
Results of the TDHF calculation: $^{40}\text{Ca}_{20}^{+}^{124}\text{Sn}_{50}^{124}$ at $E_{\text{lab}}=170$ MeV


- 3D-grid: $60 \times 60 \times 26$ (48 fm $\times$ 48 fm $\times$ 20.8 fm), Mesh size: 0.8 fm
- Skyrme force: SLy5, $\Delta t$: 0.2 fm/c, Initial separation distance: 16 fm
- Calculated impact parameter: $0 \leq b \leq 10$ fm
- Fusion reactions ($b \leq 3.95$ fm), Binary reactions ($b \geq 3.96$ fm)

Density evolution obtained from the TDHF calculation

Average number of nucleons in $V_p$

$$\langle N \rangle_{\text{PLF}} = \int_{V_p} dr \rho(r)$$

$h=3.96$ fm

$h=4.50$ fm
Results of the TDHF calculation: $^{40}\text{Ca}_{20} + ^{124}\text{Sn}_{50} \rightarrow ^{124}\text{Sn}_{124}$ at $E_{\text{lab}} = 170$ MeV


Average number of nucleons transferred from $^{124}\text{Sn}$ to $^{40}\text{Ca}$

TKEL: Total Kinetic Energy Loss

![Graph showing the average number of nucleons transferred from $^{124}\text{Sn}$ to $^{40}\text{Ca}$](image)

- Nucleons are transferred towards the direction of charge equilibrium of the system

Charge equilibration

- Neutron: $^{40}\text{Ca} \leftarrow ^{124}\text{Sn}$
- Proton: $^{40}\text{Ca} \rightarrow ^{124}\text{Sn}$

Neutron: $\text{Neutron}$ Proton: $\text{Proton}$
How to calculate the transfer probability


Particle number projection method

✓ Particle number projection operator

\[
\hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{i(n-N_P)\theta}
\]

\(\hat{N}_P\): Number operator of the spatial region \(V_P\)

\[
\hat{N}_P = \int_{V_P} d^3r \sum_{i=1}^{N_P+N_T} \delta(\mathbf{r} - \mathbf{r}_i)
\]

Projectile region: \(V_P\)

Target region: \(V_T\)

\(N = N_P + N_T\): Total number of nucleons

➢ Probability \(P_n\): \(n\) nucleons are in the \(V_P\) and \(N-n\) nucleons are in the \(V_T\)

\[
P_n = \langle \Phi | \hat{P}_n | \Phi \rangle
\]

\[
= \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{in\theta} \det \left\{ \langle \phi_i | \phi_j \rangle_{V_T} + e^{-i\theta} \langle \phi_i | \phi_j \rangle_{V_P} \right\}
\]

Slater determinant

\[
\Phi(x_1, \cdots, x_N) = \frac{1}{\sqrt{N!}} \det \{ \phi_i(x_j) \}
\]

Single-particle w.f.

\[
\phi_i(\mathbf{x}) \equiv \phi_i(\mathbf{r}, \sigma)
\]

Overlap integral in respective regions

\[
\langle \phi_i | \phi_j \rangle_{\tau} = \int_{\tau} d^3x \phi^*_i(\mathbf{x})\phi_j(\mathbf{x})
\]

\(\tau = V_P\) or \(V_T\)
Results of the TDHF calculation: $^{40}$Ca$_{20} + ^{124}$Sn$_{50}$ at $E_{\text{lab}} = 170$ MeV


Transfer probabilities

$$P_n = \langle \Phi | \hat{P}_n | \Phi \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{in\theta} \ \text{det} \{ \langle \phi_i | \phi_j \rangle_{V_T} + e^{-i\theta} \langle \phi_i | \phi_j \rangle_{V_T} \}$$

: The projection method

+n: Neutron addition

-\( n \): Neutron addition

\( \rightarrow \): Proton removal

\( \rightarrow \): Proton removal

\( 40 \)Ca \( \rightarrow \) \( 124 \)Sn

\( 40 \)Ca \( \rightarrow \) \( 124 \)Sn

Nucleons are transferred towards the directions of the charge equilibrium.

Transfer probabilities of several nucleons become sizable just outside the fusion region.
Transfer cross sections

- Horizontal axis: Number of neutrons in lighter ($^{40}$Ca-like) fragment
- Labels “(xp)”, $x=+1, \ldots, -6$: Number of protons added to (+)/removed from (-) $^{40}$Ca

Transfer cross section:

$$\sigma_{tr}(Z, N) = 2\pi \int_{b_{\text{min}}}^{\infty} b P_{Z}^{(p)}(b) P_{N}^{(n)}(b) \, db$$


Results of the TDHF calculation: $^{40}\text{Ca}_{20} + ^{124}\text{Sn}_{50} \text{ at } E_{\text{lab}} = 170 \text{ MeV}$


Projectile: $^{40}$Ca (Z=20, N=20)
Target: $^{124}$Sn (Z=50, N=74)
Possible origin of discrepancy: Particle evaporation

Effects of particle evaporation

- If produced nuclei via multinucleon transfer processes are highly-excited, these nuclei may de-excite by particle evaporation processes.

![Graph showing effects of particle evaporation](image)
Possible origin of discrepancy: Particle evaporation

- If produced nuclei via multinucleon transfer processes are highly-excited, these nuclei may de-excite by particle evaporation processes.

Particle evaporation processes may remedy the discrepancy
Possible origin of discrepancy: Particle evaporation

Effects of particle evaporation

➢ If produced nuclei via multinucleon transfer processes are highly-excited, these nuclei may de-excite by particle evaporation processes.

To evaluate the effect of particle evaporation, we need to calculate excitation energy of produced fragments for each transfer channel. We applied the particle number projection method.
How to calculate energy expectation value of produced fragments


Application of the projection method

Recall: The final w.f. is a superposition of different particle number states

Ex.) $^{40}\text{Ca} + ^{124}\text{Sn}$ reaction

$$
\Phi_{\text{final}} = C_{(0p,+1n)} \Phi(41\text{Ca};123\text{Sn}) + C_{(-1p,0n)} \Phi(39\text{K};125\text{Sb}) + C_{(-2p,0n)} \Phi(38\text{Ar};126\text{Te}) + \ldots
$$

<table>
<thead>
<tr>
<th>Projectile region: $V_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target region: $V_T$</td>
</tr>
</tbody>
</table>

Energy expectation value: $n$ nucleons are in the $V_P$ and $N-n$ nucleons are in the $V_T$

$$
E_n = \frac{\langle \Phi | \hat{H} | \hat{P}_n | \Phi \rangle}{\langle \Phi | \hat{P}_n | \Phi \rangle}
= \frac{1}{2\pi P_n} \int_0^{2\pi} d\theta e^{in\theta} \text{det} B(\theta) \int_{V_P} d^3r \tilde{\mathcal{H}}(r, \theta) + \frac{1}{2\pi P_n} \int_0^{2\pi} d\theta e^{in\theta} \text{det} B(\theta) \int_{V_T} d^3r \tilde{\mathcal{H}}(r, \theta)
$$

Energy of fragment in $V_P$ with $n$ nucleons

Energy density functional composed of $\phi_i(x, \theta)$ ($i = 1, \ldots, N_P + N_T$)

$$
\phi_i(x, \theta) = \begin{cases} 
e^{-i\theta} \phi_i(r, \sigma) & \text{if } r \in V_P \\ \phi_i(r, \sigma) & \text{if } r \notin V_P \end{cases}
$$

$$
\tilde{\phi}_i(x, \theta) = \sum_{j=1}^N \phi_j(x, \theta) (B^{-1}(\theta))_{ji}
$$

$$
\left( B(\theta) \right)_{ij} = \langle \phi_i | \phi_j \rangle_{V_T} + e^{-i\theta} \langle \phi_i | \phi_j \rangle_{V_P}
\int d^3r \tilde{\mathcal{H}}(r, \theta):
$$
Transfer cross sections

(Preliminary)

- Horizontal axis: Number of neutrons in lighter ($^{40}$Ca-like) fragment
- Labels “(xp)”, $x=+1, ..., -6$: Number of protons added to (+)/removed from (-) $^{40}$Ca

\[
\sigma_{\text{tr}}(Z, N) = 2\pi \int_{b_{\text{min}}}^{\infty} b P_Z^{(p)}(b) P_N^{(n)}(b) \, db
\]


Results of the TDHF calculation: $^{40}$Ca$_{20}$+$^{124}$Sn$_{50}$ at $E_{\text{lab}}=170$ MeV


\[ ^{40}\text{Ca} + ^{124}\text{Sn} \quad (E_{\text{lab}}=170 \text{ MeV}) \]

\[ \sigma_{\text{tr}} \quad (\text{mb}) \]

\[ \begin{array}{c}
(+1p) \\
(+0p) \\
(-1p) \\
(-2p) \\
(-3p) \\
(-4p) \\
(-5p) \\
(-6p) \\
\end{array} \]

NEUTRON NUMBER of PLF

Exp. \quad TDHF w/o evap. \quad TDHF w/ evap.

Projectile: $^{40}$Ca (Z=20, N=20)  
Target: $^{124}$Ca (Z=50, N=74)
\[ {^{48}\text{Ca}} + {^{124}\text{Sn}} \text{ reaction} \]

\[ N/Z = 1.40 \, (^{48}\text{Ca}), \, 1.48 \, (^{124}\text{Sn}) ; \, Z_p Z_T = 1000 \]
Transfer probabilities towards both directions are obtained by the projection method.

\[ P_n = \langle \Phi | \hat{P}_n | \Phi \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{in\theta} \det \{ \langle \phi_i | \phi_j \rangle_{V_T} + e^{-i\theta} \langle \phi_i | \phi_j \rangle_{V_T} \} : \text{The projection method} \]

Results of the TDHF calculation: $^{48}\text{Ca}_{28} + ^{124}\text{Sn}_{50} \rightarrow ^{124}\text{Sn}_{124} \text{ at } E_{\text{lab}} = 174 \text{ MeV}$

Results of the TDHF calculation: $^{48}\text{Ca}_{28}+^{124}\text{Sn}_{50}$ at $E_{\text{lab}}=174$ MeV

Transfer cross sections

(Preliminary)

- Horizontal axis: Number of neutrons in lighter ($^{48}\text{Ca}$-like) fragment
- Labels “(xp)”, $x=-2, ..., +2$: Number of protons added to (+)/removed from (-) $^{48}\text{Ca}$

\[
\sigma_{\text{tr}}(Z, N) = 2\pi \int_{b_{\text{min}}}^{\infty} b P_Z^p(b) P_N^n(b) \, db
\]


Projectile: $^{48}\text{Ca}$ ($Z=20$, $N=28$)
Target: $^{124}\text{Sn}$ ($Z=50$, $N=74$)
\[ ^{58}\text{Ni} + ^{208}\text{Pb} \text{ reaction} \]

\[ N/Z = 1.07 \ (^{58}\text{Ni}), \ 1.54 \ (^{208}\text{Pb}) \ ; \ Z_p Z_T = 2296 \]
Results of the TDHF calculation: $^{58}_{28}$Ni$_{30}$+$^{208}_{82}$Pb$_{126}$ at $E_{\text{lab}}$=328.4 MeV


3D-grid: $60 \times 60 \times 26$ (48 fm$\times$48 fm$\times$20.8 fm), Mesh size: 0.8 fm
Skyrme force: SLy5, $\Delta t$: 0.2 fm/c, Initial separation distance: 16 fm
Calculated impact parameter: $0 \leq b \leq 10$ fm
Fusion reactions ($b \leq 1.38$ fm), Binary reactions ($1.39$ fm $\leq b$)

Density evolution obtained from the TDHF calculation

Projectile: $^{58}$Ni
Target: $^{208}$Pb

$b=1.38$ fm

$b=1.60$ fm

$b=4.00$ fm
Results of the TDHF calculation: $^{58}_{28}$Ni$_{30} + ^{208}_{82}$Pb$_{126}$ at $E_{\text{lab}}=328.4$ MeV


3D-grid: 60×60×26 (48 fm×48 fm×20.8 fm), Mesh size: 0.8 fm
Skyrme force: SLy5, $\Delta t$: 0.2 fm/c, Initial separation distance: 16 fm
Calculated impact parameter: $0 \leq b \leq 10$ fm
Fusion reactions ($b \leq 1.38$ fm), Binary reactions ($1.39$ fm $\leq b$)

Density evolution obtained from the TDHF calculation

$b=1.38$ fm

$b=1.60$ fm

$b=4.00$ fm
Results of the TDHF calculation: \( ^{58}_{28}\text{Ni}_{30} + ^{208}_{82}\text{Pb}_{126} \) at \( E_{\text{lab}} = 328.4 \text{ MeV} \)


3D-grid: 60×60×26 (48 fm×48 fm×20.8 fm), Mesh size: 0.8 fm
Skyrme force: SLy5, \( \Delta t \): 0.2 fm/c, Initial separation distance: 16 fm
Calculated impact parameter: \( 0 \leq b \leq 10 \) fm
Fusion reactions \( (b \leq 1.38 \text{ fm}) \), Binary reactions \( (1.39 \text{ fm} \leq b) \)

Density evolution obtained from the TDHF calculation

\( b = 1.38 \text{ fm} \)

\( b = 1.60 \text{ fm} \)

\( b = 4.00 \text{ fm} \)
Results of the TDHF calculation: $^{\text{58}}_{\text{28}}\text{Ni} + ^{\text{208}}_{\text{82}}\text{Pb}$ at $E_{\text{lab}} = 328.4$ MeV


3D-grid: $60 \times 60 \times 26$ (48 fm $\times$ 48 fm $\times$ 20.8 fm), Mesh size: 0.8 fm
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Fusion reactions ($b \leq 1.38$ fm), Binary reactions ($1.39$ fm $\leq b$)

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$b = 1.38$ fm

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Density evolution obtained from the TDHF calculation

- $b = 1.38$ fm
  - Fusion ($b \leq 1.38$ fm)

- $b = 1.60$ fm
  - Transfer ($1.39$ fm $\leq b$)

- $b = 4.00$ fm
Results of the TDHF calculation: $^{58}_{28}\text{Ni}^{+} + ^{208}_{82}\text{Pb}^{126}$ at $E_{\text{lab}} = 328.4$ MeV


3D-grid: $60 \times 60 \times 26$ (48 fm $\times$ 48 fm $\times$ 20.8 fm), Mesh size: 0.8 fm

Skyrme force: SLy5, $\Delta t$: 0.2 fm/c, Initial separation distance: 16 fm

Calculated impact parameter: $0 \leq b \leq 10$ fm

Fusion reactions ($b \leq 1.38$ fm), Binary reactions ($1.39$ fm $\leq b$)

Density evolution obtained from the TDHF calculation

Average number of nucleons in $V_P$

$$\langle N \rangle_{\text{PLF}} = \int_{V_P} d\mathbf{r} \, \rho(\mathbf{r})$$

$b = 1.60$ fm

$b = 4.00$ fm
Results of the TDHF calculation: $^{58}_{28}\text{Ni}^{30}_{130} + ^{208}_{82}\text{Pb}^{126}_{126}$ at $E_{\text{lab}}=328.4$ MeV


Average number of nucleons transferred from $^{208}\text{Pb}$ to $^{58}\text{Ni}$

TKEL: Total Kinetic Energy Loss

There appear two types of transfer dynamics depending on the impact parameter:

- **At large impact parameter** ($3 \text{ fm} < b$)
  - Neutron: $^{58}_{28}\text{Ni} \leftarrow ^{208}_{82}\text{Pb}$
  - Proton: $^{58}_{28}\text{Ni} \rightarrow ^{208}_{82}\text{Pb}$

- **At small impact parameter** ($b < 3 \text{ fm}$)
  - Neutron: $^{58}_{28}\text{Ni} \leftarrow ^{208}_{82}\text{Pb}$
  - Proton: $^{58}_{28}\text{Ni} \leftarrow ^{208}_{82}\text{Pb}$

Charge equilibration

Neck breaking

$lacktriangleright$ There appear two types of transfer dynamics depending on the impact parameter
Results of the TDHF calculation: $^{58}\text{Ni}_{30} + ^{208}\text{Pb}_{126} \text{ at } E_{\text{lab}} = 328.4 \text{ MeV}$


Production cross sections for $^{58}\text{Ni}$-like fragments


- Horizontal axis: Number of neutrons in lighter ($^{58}\text{Ni}$-like) fragments
- Labels “(-x p)”, x=0, ..., 6: Number of removed protons from $^{58}\text{Ni}$

The discrepancy is somewhat remedied, but not enough.

We need to extend our theoretical framework for more realistic descriptions of MNT processes.
1. Introduction

2. TDHF calculation for MNT processes
   a) Transfer dynamics described by the TDHF theory
   b) How to calculate transfer probabilities from a TDHF wave function
   c) Systematics of MNT processes: $N/Z$ and $Z_pZ_T$ dependence

3. TDHF calculation for MNT & QF processes in reactions involving $^{238}\text{U}$

4. Summary and Perspective
Outline

1. Introduction

2. TDHF calculation for MNT processes

3. TDHF calculation for MNT & QF processes in reactions involving $^{238}$U
   
   a) $^{64}$Ni+$^{238}$U reaction: Orientations and quantum shells

   b) $^{238}$U+$^{124}$Sn reaction: Orientations and collision energies

4. Summary and Perspective
$^{64}\text{Ni} + ^{238}\text{U}$ reaction

$N/Z = 1.29 \left( ^{64}\text{Ni} \right), 1.59 \left( ^{238}\text{U} \right) ; \ Z_pZ_T = 2576$
Production cross sections for $^{64}\text{Ni}$-like fragments

- Horizontal axis: Mass number in smaller fragment (incident $^{64}\text{Ni}$)
- Labels “$(\pm x)p$”, $x=0, \ldots, 6$: Number of transferred protons

Experimental data: $^{28}_{28}\text{Ni} + ^{238}_{92}\text{U}$ at $E_{\text{lab}} = 390 \text{ MeV} \ (\sim 6.1 \text{ MeV/A})$

Production cross sections for $^{64}$Ni-like fragments

- Horizontal axis: Mass number in smaller fragment (incident $^{64}$Ni)
- Labels “$(\pm xp)$”, $x=0, \ldots, 6$: Number of transferred protons
- without effects of particle evaporation


Results of the TDHF calculation: $^{64}$Ni$_{36} + ^{238}$U$_{146}$ at $E_{\text{lab}} = 390$ MeV


K. Sekizawa

Time-dependent Hartree-Fock calculations for MNT and QF processes

Mon., 21 July, 2014 20/28

TDHF reproduces measurements reasonably, both proton stripping and pickup channels
Production cross sections for $^{64}$Ni-like fragments

- Horizontal axis: Mass number in smaller fragment (incident $^{64}$Ni)
- Labels ‘$(\pm x)p$’, $x=0$, ..., 6: Number of transferred protons
- with effects of particle evaporation


Results of the TDHF calculation: $^{64}$Ni$_{36}^{28}$ + $^{238}$U$_{146}$ at $E_{\text{lab}}=390$ MeV

TDHF reproduces measurements reasonably, both proton stripping and pickup channels
Results of the TDHF calculation: $^{64}\text{Ni}_{36} + ^{238}\text{U}_{146}$ at $E_{\text{lab}} = 390$ MeV

Production cross sections for $^{64}\text{Ni}$-like fragments


Actually, the TDHF calculation gives abundant cross sections for MNT processes from $^{238}\text{U}$ to $^{64}\text{Ni}$, up to transfer of ~50 nucleons.

K. Sekizawa
Time-dependent Hartree-Fock calculations for MNT and QF processes

Mon., 21 July, 2014
Indication of QF: $^{64}_{28}\text{Ni}_{36} + ^{238}_{92}\text{U}_{146}$ at $E_{\text{lab}} = 390$ MeV (~6.1 MeV/A)


In the article by L. Corradi et al., they mentioned:

… We observed, especially at forward angles, nuclear charges up to $Z \sim 40$, but it is difficult to get quantitative estimates of these events, since they are at the border of the spectrum and the ionization chamber was not optimized for them. We argue that these events derive from quasifission processes and from fission of $^{238}\text{U}$ (ternary events), as observed in Ref.[21].
Indication of QF: $^{64}\text{Ni}_{36} + ^{238}\text{U}_{146}$ at $E_{\text{lab}}=390$ MeV (~6.1 MeV/A)


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Ref. [21]: J. Töke et al., Nucl. Phys. A440, 327 (1985); Fig. 8 (h)
Results of the TDHF calculation: $^{64}_{28}\text{Ni}_{36} + ^{238}_{92}\text{U}_{146}$ at $E_{\text{lab}} = 390$ MeV

Production cross sections for lighter ($^{64}\text{Ni-like}$) fragments before evaporation
Results of the TDHF calculation: $^{64}_{28}\text{Ni}_{36} + ^{238}_{92}\text{U}_{146}$ at $E_{\text{lab}} = 390$ MeV

Average number of transferred nucleons

Lighter ($^{64}_{28}\text{Ni-like}$) fragment

Heavier ($^{238}_{92}\text{U-like}$) fragment

Graphs showing the change in number of neutrons and protons as a function of $b$ (fm) in the directions $x$, $y$, and $z$.
**Results of the TDHF calculation:** $^{64}_{28}\text{Ni}_{36} + ^{238}_{92}\text{U}_{146}$ at $E_{\text{lab}} = 390$ MeV

Average number of transferred nucleons

Relative orientations as well as quantum shells may affect the QF dynamics
\[ \frac{N}{Z} = 1.59 \text{ (}^{238}\text{U}), \ 1.48 \text{ (}^{124}\text{Sn}) ; \ Z_P Z_T = 4600 \]
**Objective experiment:** $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at $E_{\text{lab}} = 5.7$ MeV/A


Production cross sections for lighter ($^{124}\text{Sn}$-like) fragments
Objective experiment: \( ^{238}_{92}U_{146} + ^{124}_{50}Sn_{74} \) at \( E_{\text{lab}} = 5.7 \text{ MeV/A} \)


Production cross sections for lighter \((^{124}_{50}\text{Sn-like})\) fragments

\( ^{238}_{92}U \rightarrow ^{124}_{50}\text{Sn} \)

Neutron \(\sim 10\) transferred
Proton \(\sim 15\) transferred
Objective experiment: $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at $E_{\text{lab}} = 5.7$ MeV/A


Production cross sections for lighter ($^{124}\text{Sn}$-like) fragments

$^{238}\text{U} \rightarrow ^{124}\text{Sn}$
Neutron $\sim 10$ transferred
Proton $\sim 15$ transferred

How can we understand this massive transfer from $^{238}\text{U}$ to $^{124}\text{Sn}$ through TDHF calculation?
Production cross sections for lighter \((^{124}_{50}\text{Sn-like})\) fragments before evaporation

Results of the TDHF calculation: \(^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74} \) at \(E_{\text{lab}} = 5.7 \text{ MeV/A}\)
Production cross sections for lighter ($^{124}$Sn-like) fragments

Results of the TDHF calculation: $^{238}_{92}$U$_{146} + ^{124}_{50}$Sn$_{74}$ at $E_{\text{lab}} = 5.7$ MeV/A

- $x$-direction
- $y$-direction
- $z$-direction

Results of the TDHF calculation: $^{238\text{U}_{146}} + ^{124\text{Sn}_{74}}$ at $E_{\text{lab}} = 5.7$ MeV/A

Production cross sections for lighter ($^{124}\text{Sn-like}$) fragments after evaporation

$\sigma$ (mb)

$x$-direction

$y$-direction

$z$-direction

$\sigma$ (mb)

$A$

$Z$

$100$ $120$ $140$ $160$

$100$ $120$ $140$ $160$

$100$ $120$ $130$ $140$ $150$

Exp.

$80\mu$b

$160\mu$b

$320\mu$b

$640\mu$b

$1280\mu$b

$N=82$

$\text{K. Sekizawa}$

Time-dependent Hartree-Fock calculations for MNT and QF processes

Mon., 21 July, 2014

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➢ Measured proton flow might be contributed from the “x-direction” configuration with the neck breaking dynamics
Results of the TDHF calculation: $^{238}_{92}U_{146} + ^{124}_{50}Sn_{74}$ at different $E_{lab}$ ($b=0$)

Collision energy dependence of the reaction dynamics

$E_{lab} \sim 6$ MeV/A

$y$-direction

$E_{lab} \sim 9$ MeV/A

$y$-direction

$x$-direction
Results of the TDHF calculation: $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at different $E_{\text{lab}}$ ($b=0$)

Collision energy dependence of the reaction dynamics

For $E_{\text{lab}} \sim 6\text{ MeV/A}$

- $y$-direction
- $x$-direction

For $E_{\text{lab}} \sim 9\text{ MeV/A}$

- $y$-direction
- $x$-direction
Results of the TDHF calculation: $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at different $E_{\text{lab}}$ ($b=0$)

Collision energy dependence of the reaction dynamics

$E_{\text{lab}} \sim 6 \text{ MeV/A}$

- $y$-direction: # of transferred nucleons is very small
- $x$-direction

$E_{\text{lab}} \sim 9 \text{ MeV/A}$

- $y$-direction
- $x$-direction
Results of the TDHF calculation: \( ^{238}_{92}U_{146} + ^{124}_{50}Sn_{74} \) at different \( E_{\text{lab}} \) (\( b=0 \))

Collision energy dependence of the reaction dynamics

\[ E_{\text{lab}} \sim 6 \text{ MeV}/A \]

\( y \)-direction

\# of transferred nucleons is very small

\[ E_{\text{lab}} \sim 9 \text{ MeV}/A \]

\( y \)-direction

\[ x \]-direction

K. Sekizawa

Time-dependent Hartree-Fock calculations for MNT and QF processes

Mon., 21 July, 2014
Results of the TDHF calculation: \( \frac{^{238}_{92}U}{^{146}_{50}Sn} \) at different \( E_{\text{lab}} \) (\( b=0 \))

Collision energy dependence of the reaction dynamics

\begin{align*}
E_{\text{lab}} & \sim 6 \text{ MeV/A} \\
y\text{-direction} & \quad 6 \text{ protons and 10 neutrons} \\
\text{# of transferred nucleons is very small} \\
\end{align*}

\begin{align*}
E_{\text{lab}} & \sim 9 \text{ MeV/A} \\
y\text{-direction} \\
\text{x-direction} \\
\end{align*}
Results of the TDHF calculation: $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at different $E_{\text{lab}}$ ($b=0$)

Collision energy dependence of the reaction dynamics

$E_{\text{lab}} \approx 6$ MeV/A

- **y-direction**
  - # of transferred nucleons is very small

- **x-direction**
  - 6 protons and 10 neutrons

$E_{\text{lab}} \approx 9$ MeV/A

- **y-direction**

- **x-direction**
Results of the TDHF calculation: $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at different $E_{\text{lab}} (b=0)$

Collision energy dependence of the reaction dynamics

$E_{\text{lab}} \sim 6 \text{ MeV}/A$

\begin{align*}
\text{y-direction} & \quad \text{x-direction} \\
\text{# of transferred nucleons is very small} & \quad 6 \text{ protons and 10 neutrons}
\end{align*}

$E_{\text{lab}} \sim 9 \text{ MeV}/A$

\begin{align*}
\text{y-direction} & \quad \text{x-direction} \\
\text{# of transferred nucleons is very small} & \quad 6 \text{ protons and 10 neutrons}
\end{align*}
Results of the TDHF calculation: \( ^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74} \) at different \( E_{\text{lab}} \) (\( b=0 \))

Collision energy dependence of the reaction dynamics

**\( E_{\text{lab}} \sim 6 \text{ MeV/A} \)**

- **y-direction**
  - # of transferred nucleons is very small

- **x-direction**
  - 6 protons and 10 neutrons

**\( E_{\text{lab}} \sim 9 \text{ MeV/A} \)**

- **y-direction**
  - # of transferred nucleons is very small

- **x-direction**
Results of the TDHF calculation: $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at different $E_{\text{lab}} (b=0)$

Collision energy dependence of the reaction dynamics

**$E_{\text{lab}} \sim 6 \text{ MeV/A}$**

- **$y$-direction**
  - # of transferred nucleons is very small

- **$x$-direction**
  - 6 protons and 10 neutrons

**$E_{\text{lab}} \sim 9 \text{ MeV/A}$**

- **$y$-direction**
  - # of transferred nucleons is very small

- **$x$-direction**
  - 12 protons and 16 neutrons
Results of the TDHF calculation: $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at different $E_{\text{lab}}$ ($b=0$)

Collision energy dependence of the reaction dynamics

$E_{\text{lab}} \sim 6$ MeV/A

$y$-direction

# of transferred nucleons is very small

$x$-direction

6 protons and 10 neutrons

$E_{\text{lab}} \sim 9$ MeV/A

$y$-direction

# of transferred nucleons is very small

$x$-direction

12 protons and 16 neutrons

QF dynamics depends much not only on relative orientations but also on the collision energy
Results of the TDHF calculation: $^{238}_{92}\text{U}_{146} + ^{124}_{50}\text{Sn}_{74}$ at different $E_{\text{lab}}$ ($b=0$)

Average number of nucleons in $^{238}\text{U}$-like fragment

- **Neutron (a)**
- **Proton (b)**
  - $x$-direction
  - $y$-direction

Average number of nucleons in $^{124}\text{Sn}$-like fragment

- **Neutron (a)**
- **Proton (b)**
  - $x$-direction
  - $y$-direction
Outline

1. Introduction

2. TDHF calculation for MNT processes

3. TDHF calculation for MNT & QF processes in reactions involving $^{238}$U

4. Summary and Perspective
Summary

✔ I showed how to calculate transfer probabilities from the TDHF wave function.

✔ I presented results of the TDHF calculation for \( ^{40,48}\text{Ca}^{124}\text{Sn} \) and \( ^{58}\text{Ni}^{208}\text{Pb} \) reactions.
   (K.S. and K. Yabana, PRC88(2013)014614)

✔ Recent applications for reactions involving \( ^{238}\text{U} \) were presented. (\( ^{64}\text{Ni}^{238}\text{U}, ^{238}\text{U}^{124}\text{Sn} \))
Summary

✔ I showed how to calculate transfer probabilities from the TDHF wave function.  

✔ I presented results of the TDHF calculation for $^{40,48}\text{Ca} + ^{124}\text{Sn}$ and $^{58}\text{Ni} + ^{208}\text{Pb}$ reactions.  
        (K.S. and K. Yabana, PRC 88 (2013) 014614)

✔ Recent applications for reactions involving $^{238}\text{U}$ were presented. ($^{64}\text{Ni} + ^{238}\text{U}$, $^{238}\text{U} + ^{124}\text{Sn}$)

Perspective

✔ Perform a lot of calculations to understand reaction mechanisms of MNT & QF processes.

✔ Extension of the study: TDRPA, TDGCM, TDHFB (or TDHF+BCS), other projections, ...
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Thank you for your attention.