Transfer dynamics in the TDHF theory deduced from particle-number projection method

Kazuyuki Sekizawa

(Univ. Tsukuba, Japan)

In collaboration with

Kazuhiro Yabana

Graduate School of Pure and Applied Sciences, Univ. Tsukuba
Center for Computational Sciences, Univ. Tsukuba
INTRODUCTION: Interests for multinucleon transfer (MNT) reactions
INTRODUCTION: Interests for multinucleon transfer (MNT) reactions

➢ Our interests:
Our interests:

MNT reaction = a non-equilibrium quantum transport phenomenon
Our interests:

MNT reaction = a non-equilibrium quantum transport phenomenon

Static properties
- orbitals
- Fermi energies
- deformation

Dynamical properties
- vibrations
- deformation
- nucleon exchanges
Our interests:

MNT reaction = a non-equilibrium quantum transport phenomenon
INTRODUCTION: Interests for multinucleon transfer (MNT) reactions

➢ Our interests:

MNT reaction = a non-equilibrium quantum transport phenomenon

Mutually related

Static properties

- orbitals
- Fermi energies
- deformation

\[ \begin{array}{c}
  n \\
  p \\
\end{array} \]

Dynamical properties

- vibrations
- deformation
- nucleon exchanges

Aims:

➢ To understand microscopic reaction mechanism of the MNT reaction
➢ To predict preferable conditions to produce objective unstable nuclei
Current status and future plan of our study

Aims:
➢ To understand microscopic reaction mechanism of the MNT reaction
➢ To predict preferable conditions to produce objective unstable nuclei
We showed that TDHF theory can describe MNT cross sections quantitatively in an accuracy comparable to existing theories.


Aims:
- To understand microscopic reaction mechanism of the MNT reaction
- To predict preferable conditions to produce objective unstable nuclei
Current status and future plan of our study

Aims:
- To understand microscopic reaction mechanism of the MNT reaction
- To predict preferable conditions to produce objective unstable nuclei

We showed that TDHF theory can describe MNT cross sections quantitatively in an accuracy comparable to existing theories.


1. Systematic TDHF simulations for various initial conditions (projectile-target, $E_{\text{lab}}$, $b$)
Current status and future plan of our study

Aims:

- To understand microscopic reaction mechanism of the MNT reaction
- To predict preferable conditions to produce objective unstable nuclei

✔ We showed that TDHF theory can describe MNT cross sections quantitatively in an accuracy comparable to existing theories.


1. Systematic TDHF simulations for various initial conditions (projectile-target, $E_{\text{lab}}$, $b$)

2. Extension to the TDHFB to examine effects of pairing correlations on reaction dynamics by Y. Hashimoto, in progress
We showed that TDHF theory can describe MNT cross sections quantitatively in an accuracy comparable to existing theories.


Aims:

- To understand microscopic reaction mechanism of the MNT reaction
- To predict preferable conditions to produce objective unstable nuclei

1. Systematic TDHF simulations for various initial conditions (projectile-target, $E_{\text{lab}}$, $b$)

2. Extension to the TDHFB to examine effects of pairing correlations on reaction dynamics by Y. Hashimoto, in progress

3. Develop methods of analysis to obtain more information on reaction mechanisms
Current status and future plan of our study

Aims:
- To understand microscopic reaction mechanism of the MNT reaction
- To predict preferable conditions to produce objective unstable nuclei

We showed that TDHF theory can describe MNT cross sections quantitatively in an accuracy comparable to existing theories.


1. Systematic TDHF simulations for various initial conditions (projectile-target, $E_{\text{lab}}, b$)

2. Extension to the TDHFB to examine effects of pairing correlations on reaction dynamics by Y. Hashimoto, in progress

3. Develop methods of analysis to obtain more information on reaction mechanisms
Current status and future plan of our study

Aims:

- To understand microscopic reaction mechanism of the MNT reaction
- To predict preferable conditions to produce objective unstable nuclei

We showed that TDHF theory can describe MNT cross sections quantitatively in an accuracy comparable to existing theories.


Today’s topic

1. Systematic TDHF simulations for various initial conditions (projectile-target, $E_{lab}$, $b$)

2. Extension to the TDHFB to examine effects of pairing correlations on reaction dynamics by Y. Hashimoto, in progress

3. Develop methods of analysis to obtain more information on reaction mechanisms
1. Introduction

2. Idea: to analyze reaction products in TDHF with PNP

3. An illustrative example: $^{24}$O+$^{16}$O reactions

4. Summary and Perspective
1. Introduction

2. Idea: to analyze reaction products in TDHF with PNP

3. An illustrative example: $^{24}\text{O} + ^{16}\text{O}$ reactions

4. Summary and Perspective
Our approach: The time-dependent Hartree-Fock (TDHF) theory

- Real space: 3D box discretized into a uniform mesh
- Real time: Taylor expansion method of 4th order
- Contact type (Skyrme) effective interaction
- No adjustable parameter specific to the reaction dynamics

We solve Skyrme TDHF equation in real space and real time

\[
\frac{\delta}{\delta \phi_i^*} \left[ \int dt \left\langle \Phi \left| i\hbar \frac{\partial}{\partial t} - \hat{H}_{\text{Skyrme}} \right| \Phi \right\rangle \right] = 0
\]

Variation

\[
i\hbar \frac{\partial \phi_i(r, \sigma, q, t)}{\partial t} = \hat{h}_{\text{Skyrme}} [\rho, \tau, \mathbf{j}, \mathbf{s}, \mathbf{T}, J_{\mu\nu}] \phi_i(r, \sigma, q, t) : \text{TDHF eq.}
\]

Slater determinant:

\[
\Phi(x_1, \ldots, x_N, t) = \frac{1}{\sqrt{N!}} \det \{ \phi_i(x_j, t) \}
\]

Single-particle wave function:

\[
\phi_i(r, \sigma, q, t) \equiv \phi_i(x, t)
\]
What do ordinary expectation values tell us?
What do ordinary expectation values tell us?

Example: $^{24}\text{O} + ^{16}\text{O}$ reaction at $E_{\text{lab}} = 4$ MeV/A, $b = 7.1$ fm
What do ordinary expectation values tell us?

Example: $^{24}\text{O} + ^{16}\text{O}$ reaction at $E_{\text{lab}}=4$ MeV/A, $b=7.1$ fm
Example: $^{24}\text{O}+^{16}\text{O}$ reaction at $E_{\text{lab}}=4$ MeV/A, $b=7.1$ fm
What do ordinary expectation values tell us?

Example: $^{24}\text{O} + ^{16}\text{O}$ reaction at $E_{\text{lab}} = 4 \text{ MeV/A}, b = 7.1 \text{ fm}

**Number operator in a spatial region $V$**

$$\hat{N}_V^{(q)} = \int_V d\mathbf{r} \sum_{i \in q} \delta(\mathbf{r} - \mathbf{r}_i)$$

**Expectation value of $\hat{N}_V^{(q)}$**

$$\langle \hat{N}_V^{(q)} \rangle \equiv \langle \Phi | \hat{N}_V^{(q)} | \Phi \rangle = \int_V d\mathbf{r} \, \rho(\mathbf{r})$$
What do ordinary expectation values tell us?

Example: $^{24}$O+$^{16}$O reaction at $E_{\text{lab}}=4$ MeV/A, $b=7.1$ fm

Number operator in a spatial region $V$

$$\hat{N}^{(q)}_V = \int_V d\mathbf{r} \sum_{i \in q} \delta(\mathbf{r} - \mathbf{r}_i)$$

Expectation value of $\hat{N}^{(q)}_V$

$$\langle \hat{N}^{(q)}_V \rangle \equiv \langle \Phi | \hat{N}^{(q)}_V | \Phi \rangle = \int_V d\mathbf{r} \rho(\mathbf{r})$$

$$\langle \hat{N}^{(q)}_{V_P} \rangle = 14.697 \text{ neutron}$$
$$8.643 \text{ proton}$$

$$\langle \hat{N}^{(q)}_{V_T} \rangle = 9.127 \text{ neutron}$$
$$7.353 \text{ proton}$$

$$\langle \hat{N}^{(q)}_{V_{out}} \rangle = 0.176 \text{ neutron}$$
$$0.004 \text{ proton}$$
What do ordinary expectation values tell us?

Example: $^{24}\text{O} + ^{16}\text{O}$ reaction at $E_{\text{lab}} = 4 \text{ MeV/A}$, $b = 7.1 \text{ fm}$

Number operator in a spatial region $V$

$$\hat{N}_V^{(q)} = \int_V d\mathbf{r} \sum_{i \in q} \delta(\mathbf{r} - \mathbf{r}_i)$$

Expectation value of $\hat{N}_V^{(q)}$

$$\langle \hat{N}_V^{(q)} \rangle \equiv \langle \Phi | \hat{N}_V^{(q)} | \Phi \rangle = \int_V d\mathbf{r} \rho(\mathbf{r})$$

Transfer dynamics in the TDHF theory deduced from particle-number projection method

$$\begin{align*}
\langle \hat{N}_{V_{P}}^{(n)} \rangle &= 14.697 \text{ neutron} \\
&\quad 8.643 \text{ proton} \\
\langle \hat{N}_{V_{T}}^{(n)} \rangle &= 9.127 \text{ neutron} \\
&\quad 7.353 \text{ proton} \\
\langle \hat{N}_{V_{\text{out}}}^{(n)} \rangle &= 0.176 \text{ neutron} \\
&\quad 0.004 \text{ proton}
\end{align*}$$

Neutron: $^{16}\text{O} \leftrightarrow ^{24}\text{O}$
Proton: $^{16}\text{O} \rightarrow ^{24}\text{O}$
What do ordinary expectation values tell us?

Example: $^{24}\text{O} + ^{16}\text{O}$ reaction at $E_{\text{lab}} = 4$ MeV/A, $b = 7.1$ fm

✔ From ordinary expectation values, we may get qualitative understandings of the reaction.

\[
\langle \hat{N}_{VP} \rangle = \begin{align*}
14.697 & \quad \text{neutron} \\
8.643 & \quad \text{proton}
\end{align*}
\]

\[
\langle \hat{N}_{VT} \rangle = \begin{align*}
9.127 & \quad \text{neutron} \\
7.353 & \quad \text{proton}
\end{align*}
\]

\[
\langle \hat{N}_{V_{out}} \rangle = \begin{align*}
0.176 & \quad \text{neutron} \\
0.004 & \quad \text{proton}
\end{align*}
\]

Neutron: $^{16}\text{O} \leftarrow ^{24}\text{O}$
Proton: $^{16}\text{O} \rightarrow ^{24}\text{O}$
What do ordinary expectation values tell us?

Example: $^{24}\text{O} + ^{16}\text{O}$ reaction at $E_{\text{lab}} = 4 \text{ MeV/A}, b = 7.1 \text{ fm}$

✔ From ordinary expectation values, we may get qualitative understandings of the reaction.

How can we get more information?

\[ \langle \hat{N}_{V_P}^{(q)} \rangle = 14.697 \text{ neutron} \]
\[ \langle \hat{N}_{V_T}^{(q)} \rangle = 8.643 \text{ proton} \]

\[ \langle \hat{N}_{V_{out}}^{(q)} \rangle = 9.127 \text{ neutron} \]
\[ \langle \hat{N}_{V_{out}}^{(q)} \rangle = 7.353 \text{ proton} \]

\[ \langle \hat{N}_{V_{out}}^{(q)} \rangle = 0.176 \text{ neutron} \]
\[ \langle \hat{N}_{V_{out}}^{(q)} \rangle = 0.004 \text{ proton} \]

Neutron: $^{16}\text{O} \leftrightarrow ^{24}\text{O}$
Proton: $^{16}\text{O} \rightarrow ^{24}\text{O}$

Example: $^{24}\text{O}^+ + ^{16}\text{O}$ reaction at $E_{\text{lab}} = 4 \text{ MeV/A}, b = 7.1 \text{ fm}$

Particle-number projection technique

\[ \hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \, e^{i(n-\hat{N}_P)\theta} \]

\[ \hat{N}_P: \text{Number operator of the spatial region } V_P \]

\[ \hat{N}_P = \int_{V_P} d^3r \sum_{i=1}^{N_P+N_T} \delta(\mathbf{r} - \mathbf{r}_i) \]

\( N = N_P + N_T: \text{Total number of nucleons} \)
Probability $P_n$: $n$ nucleons are in the projectile region $V_P$

\[
P_n = \langle \Phi_n | \Phi_n \rangle = \langle \Phi | \hat{P}_n | \Phi \rangle
= \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{in\theta} \det \left\{ \langle \phi_i | \phi_j \rangle_{V_T} + e^{-i\theta} \langle \phi_i | \phi_j \rangle_{V_P} \right\}
\]

Particle-number projection operator

\[
\hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{i(n-\hat{N}_P)\theta}
\]

\[
\hat{N}_P: \text{Number operator of the spatial region } V_P
\]

\[
\hat{N}_P = \int_{V_P} d^3r \ \sum_{i=1}^{N_P+N_T} \delta(\mathbf{r} - \mathbf{r}_i)
\]

Slater determinant

\[
\Phi(\mathbf{x}_1, \ldots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \det \left\{ \phi_i(\mathbf{x}_j) \right\}
\]

Single-particle w.f.s

\[
\phi_i(\mathbf{x}) \equiv \phi_i(\mathbf{r}, \sigma)
\]

Overlap integral in each region

\[
\langle \phi_i | \phi_j \rangle_{\tau} = \int_{\tau} d^3x \ \phi_i^*(\mathbf{x})\phi_j(\mathbf{x})
\]

$\tau = V_P$ or $V_T$

$N = N_P + N_T$: Total number of nucleons
Particle-number projection technique


✔ Particle number projection operator

\[ \hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{i(n-\hat{N}_P)\theta} \]

\[ \hat{N}_P: \text{Number operator of the spatial region } V_P \]

\[ \hat{N}_P = \int_{V_P} d^3r \sum_{i=1}^{N_P+N_T} \delta(r - r_i) \]

➢ Probability \( P_n \): \( N \) neutrons are in the \( V_P \); \( Z \) protons are in the \( V_P \)
Because we have the TDHF wave function after collision, in principle, we can get more information about the reaction

\[ |\Phi_n\rangle \equiv \hat{P}_n |\Phi\rangle \] : Particle-number projected wave function

We try to analyze the particle-number projected w.f.

- **Probability** $P_n$: $N$ neutrons are in the $V_P$; $Z$ protons are in the $V_P$
Definition of operators for a fragment nucleus

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
Definition of operators for a fragment nucleus

Space division function

\[ \Theta_V(r) = \begin{cases} 
1 & \text{if } r \in V \\
0 & \text{if } r \notin V 
\end{cases} \]

Spatial region \( V \): includes a fragment to be analyzed

Spatial region \( \bar{V} \): the complement of \( V \)

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
**Definition of operators for a fragment nucleus**

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

### Space division function

\[ \Theta_V(r) = \begin{cases} 
1 & \text{if } r \in V \\
0 & \text{if } r \notin V 
\end{cases} \]

### Spatial region

- **\( V \):** includes a fragment to be analyzed
- **\( \overline{V} \):** the complement of **\( V \)**

### One-body operator

\[ \hat{\mathcal{O}}^{(1)} = \sum_{i=1}^{N} \hat{\mathcal{O}}^{(1)}(r_i, \sigma_i) \]
Definition of operators for a fragment nucleus

Space division function

\[ \Theta_V(\mathbf{r}) = \begin{cases} 
1 & \text{if } \mathbf{r} \in V 
\0 & \text{if } \mathbf{r} \notin V 
\end{cases} \]

Spatial region $V$: includes a fragment to be analyzed
Spatial region $\overline{V}$: the complement of $V$

One-body operator

\[ \hat{\mathcal{O}}^{(1)} = \sum_{i=1}^{N} \hat{\mathcal{O}}^{(1)}(\mathbf{r}_i, \sigma_i) = \sum_{i=1}^{N} \left[ \Theta_V(\mathbf{r}_i) + \Theta_{\overline{V}}(\mathbf{r}_i) \right] \hat{\mathcal{O}}^{(1)}(\mathbf{r}_i, \sigma_i) \]

= 1

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
**Definition of operators for a fragment nucleus**

**Space division function**

\[ \Theta_V(r) = \begin{cases} 
1 & \text{if } r \in V \\
0 & \text{if } r \notin V 
\end{cases} \]

**Spatial region** $V$: includes a fragment to be analyzed

**Spatial region** $\overline{V}$: the complement of $V$

**One-body operator**

\[
\hat{O}^{(1)} = \sum_{i=1}^{N} \hat{\rho}^{(1)}(r_i, \sigma_i) = \sum_{i=1}^{N} \left[ \Theta_V(r_i) + \Theta_{\overline{V}}(r_i) \right] \hat{\rho}^{(1)}(r_i, \sigma_i)
\]

\[
= \hat{O}_V^{(1)} + \hat{O}_{\overline{V}}^{(1)}
\]

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
Definition of operators for a fragment nucleus

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

Space division function

\[ \Theta_V(r) = \begin{cases} 
1 & \text{if } r \in V \\
0 & \text{if } r \notin V 
\end{cases} \]

Spatial region \( V \): includes a fragment to be analyzed
Spatial region \( \overline{V} \): the complement of \( V \)

One-body operator

\[ \hat{\mathcal{O}}^{(1)} = \sum_{i=1}^{N} \hat{o}^{(1)}(r_i, \sigma_i) = \sum_{i=1}^{N} \left[ \Theta_V(r_i) + \Theta_{\overline{V}}(r_i) \right] \hat{o}^{(1)}(r_i, \sigma_i) = 1 \]

Two-body operator

\[ \hat{\mathcal{O}}^{(2)} = \sum_{i<j} \hat{o}^{(2)}(r_i, \sigma_i, r_j, \sigma_j) \]
**Definition of operators for a fragment nucleus**

Space division function

\[
\Theta_{V}(\mathbf{r}) = \begin{cases} 
1 & \text{if } \mathbf{r} \in V \\
0 & \text{if } \mathbf{r} \notin V
\end{cases}
\]

One-body operator

\[
\hat{O}^{(1)} = \sum_{i=1}^{N} \hat{o}^{(1)}(\mathbf{r}_i, \sigma_i) = \sum_{i=1}^{N} \left[ \Theta_{V}(\mathbf{r}_i) + \Theta_{\bar{V}}(\mathbf{r}_i) \right] \hat{o}^{(1)}(\mathbf{r}_i, \sigma_i) 
= \hat{O}_{V}^{(1)} + \hat{O}_{\bar{V}}^{(1)}
\]

Two-body operator

\[
\hat{O}^{(2)} = \sum_{i<j} \hat{o}^{(2)}(\mathbf{r}_i, \sigma_i, \mathbf{r}_j, \sigma_j) = \sum_{i<j} \left[ \Theta_{V}(\mathbf{r}_i) + \Theta_{\bar{V}}(\mathbf{r}_i) \right] \left[ \Theta_{V}(\mathbf{r}_j) + \Theta_{\bar{V}}(\mathbf{r}_j) \right] \hat{o}^{(2)}(\mathbf{r}_i, \sigma_i, \mathbf{r}_j, \sigma_j) 
= \hat{O}_{V}^{(2)} + \hat{O}_{\bar{V}}^{(2)}
= 1
\]
**Definition of operators for a fragment nucleus**

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

Space division function

\[
\Theta_V(r) = \begin{cases} 
1 & \text{if } r \in V \\
0 & \text{if } r \notin V 
\end{cases}
\]

Spatial region \(V\): includes a fragment to be analyzed

Spatial region \(\overline{V}\): the complement of \(V\)

One-body operator

\[
\hat{\mathcal{O}}^{(1)} = \sum_{i=1}^{N} \hat{\mathcal{O}}^{(1)}(r_i, \sigma_i) = \sum_{i=1}^{N} \left[ \Theta_V(r_i) + \Theta_{\overline{V}}(r_i) \right] \hat{\mathcal{O}}^{(1)}(r_i, \sigma_i) = 1
\]

\[
= \hat{\mathcal{O}}^{(1)}_V + \hat{\mathcal{O}}^{(1)}_{\overline{V}}
\]

Two-body operator

\[
\hat{\mathcal{O}}^{(2)} = \sum_{i<j}^{N} \hat{\mathcal{O}}^{(2)}(r_i, \sigma_i, r_j, \sigma_j) = \sum_{i<j}^{N} \left[ \Theta_V(r_i) + \Theta_{\overline{V}}(r_i) \right] \left[ \Theta_V(r_j) + \Theta_{\overline{V}}(r_j) \right] \hat{\mathcal{O}}^{(2)}(r_i, \sigma_i, r_j, \sigma_j) = 1
\]

\[
= \sum_{i<j}^{N} \left[ \Theta_V(r_i)\Theta_V(r_j) + \Theta_{\overline{V}}(r_i)\Theta_{\overline{V}}(r_j) + \Theta_V(r_i)\Theta_{\overline{V}}(r_j) + \Theta_{\overline{V}}(r_i)\Theta_V(r_j) \right] \hat{\mathcal{O}}^{(2)}(r_i, \sigma_i, r_j, \sigma_j)
\]
Definition of operators for a fragment nucleus

Space division function

\[ \Theta_V(r) = \begin{cases} 
1 & \text{if } r \in V \\
0 & \text{if } r \notin V
\end{cases} \]

Spatial region \( V \): includes a fragment to be analyzed
Spatial region \( \overline{V} \): the complement of \( V \)

One-body operator

\[
\hat{O}^{(1)} = \sum_{i=1}^{N} \hat{O}^{(1)}(r_i, \sigma_i) = \sum_{i=1}^{N} \left[ \Theta_V(r_i) + \Theta_{\overline{V}}(r_i) \right] \hat{O}^{(1)}(r_i, \sigma_i)
= \hat{O}_V^{(1)} + \hat{O}_{\overline{V}}^{(1)} = 1
\]

Two-body operator

\[
\hat{O}^{(2)} = \sum_{i<j} \hat{O}^{(2)}(r_i, \sigma_i, r_j, \sigma_j) = \sum_{i<j} \left[ \Theta_V(r_i) + \Theta_{\overline{V}}(r_i) \right] \left[ \Theta_V(r_j) + \Theta_{\overline{V}}(r_j) \right] \hat{O}^{(2)}(r_i, \sigma_i, r_j, \sigma_j)
= 1 = 1
= \sum_{i<j} \left[ \Theta_V(r_i)\Theta_V(r_j) + \Theta_V(r_i)\Theta_{\overline{V}}(r_j) + \Theta_V(r_i)\Theta_{\overline{V}}(r_j) + \Theta_{\overline{V}}(r_i)\Theta_V(r_j) \right] \hat{O}^{(2)}(r_i, \sigma_i, r_j, \sigma_j)
= \hat{O}_V^{(2)} + \hat{O}_{\overline{V}}^{(2)} + \hat{O}_{V\overline{V}}^{(2)}
\]
Definition of operators for a fragment nucleus

Space division function

\[
\Theta_V (\mathbf{r}) = \begin{cases} 
1 & \text{if } \mathbf{r} \in V \\
0 & \text{if } \mathbf{r} \notin V
\end{cases}
\]

One-body operator

\[
\hat{\mathcal{O}}^{(1)} = \sum_{i=1}^{N} \hat{\mathcal{O}}^{(1)}(\mathbf{r}_i, \sigma_i) = \sum_{i=1}^{N} \left[ \Theta_V (\mathbf{r}_i) + \Theta_{\overline{V}} (\mathbf{r}_i) \right] \hat{\mathcal{O}}^{(1)}(\mathbf{r}_i, \sigma_i) = 1
\]

Two-body operator

\[
\hat{\mathcal{O}}^{(2)} = \sum_{i<j} \hat{\mathcal{O}}^{(2)}(\mathbf{r}_i, \sigma_i, \mathbf{r}_j, \sigma_j) = \sum_{i<j} \left[ \Theta_V (\mathbf{r}_i) + \Theta_{\overline{V}} (\mathbf{r}_i) \right] \left[ \Theta_V (\mathbf{r}_j) + \Theta_{\overline{V}} (\mathbf{r}_j) \right] \hat{\mathcal{O}}^{(2)}(\mathbf{r}_i, \sigma_i, \mathbf{r}_j, \sigma_j) = 1
\]

Spatial region \( V \): includes a fragment to be analyzed

Spatial region \( \overline{V} \): the complement of \( V \)
Space division function

\[ \Theta_V(\mathbf{r}) = \begin{cases} 
1 & \text{if } \mathbf{r} \in V \\
0 & \text{if } \mathbf{r} \notin V 
\end{cases} \]

\[ \hat{\mathcal{O}}_V : \text{operator for a fragment which is included in the spatial region } V \]

Spatial region \( V \): includes a fragment to be analyzed

Spatial region \( \overline{V} \): the complement of \( V \)

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
**Expectation value of operators in a particle-number projected w.f.**

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

Space division function

$$\Theta_V(r) = \begin{cases} 
1 & \text{if } r \in V \\
0 & \text{if } r \notin V 
\end{cases}$$

Spatial region $V$: includes a fragment to be analyzed

Spatial region $\overline{V}$: the complement of $V$

$\hat{O}_V$: operator for a fragment which is included in the spatial region $V$

The expectation value of the operator $\hat{O}$ in the fragment nucleus included in the spatial region $V$ composed of $n$ nucleons would be given by

$$\mathcal{O}_n^V = \frac{\langle \Phi_n | \hat{O}_V | \Phi_n \rangle}{\langle \Phi_n | \Phi_n \rangle} = \frac{\langle \Phi | \hat{O}_V \hat{P}_n | \Phi \rangle}{\langle \Phi | \hat{P}_n | \Phi \rangle}$$

$$= \frac{1}{2\pi P_n} \int_0^{2\pi} d\theta \ e^{in\theta} \langle \Phi | \hat{O}_V e^{-i\hat{N}_V \theta} | \Phi \rangle$$

Particle-number projected w.f.

$$| \Phi_n \rangle \equiv \hat{P}_n | \Phi \rangle$$

PNP operator

$$\hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{i(n-\hat{N}_V)\theta}$$
Space division function

$$\Theta_V (\mathbf{r}) = \begin{cases} 1 & \text{if } \mathbf{r} \in V \\ 0 & \text{if } \mathbf{r} \notin V \end{cases}$$

$\hat{\mathcal{O}}_V$ : operator for a fragment which is included in the spatial region $V$

The expectation value of the operator $\hat{\mathcal{O}}$ in the fragment nucleus included in the spatial region $V$ composed of $n$ nucleons would be given by

$$\mathcal{O}_n^V = \frac{\langle \Phi_n | \hat{\mathcal{O}}_V | \Phi_n \rangle}{\langle \Phi_n | \Phi_n \rangle} = \frac{\langle \Phi | \hat{\mathcal{O}}_V \hat{P}_n | \Phi \rangle}{\langle \Phi | \hat{P}_n | \Phi \rangle}$$

$$= \frac{1}{2\pi P_n} \int_0^{2\pi} d\theta \ e^{in\theta} \langle \Phi | \hat{\mathcal{O}}_V e^{-i\hat{N}_V \theta} | \Phi \rangle$$

Particle-number projected w.f.

$$|\Phi_n\rangle \equiv \hat{P}_n |\Phi\rangle$$

PNP operator

$$\hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{i(n-\hat{N}_V)\theta}$$
The expectation value of the operator $\hat{O}$ in the fragment nucleus included in the spatial region $V$ composed of $n$ nucleons would be given by

$$O^V_n = \frac{\langle \Phi_n | \hat{O}_V \hat{P}_n | \Phi_n \rangle}{\langle \Phi_n | \Phi_n \rangle} = \frac{\langle \Phi | \hat{O}_V \hat{P}_n | \Phi \rangle}{\langle \Phi | \hat{P}_n | \Phi \rangle},$$

$$= \frac{1}{2\pi P_n} \int_0^{2\pi} d\theta \ e^{in\theta} \langle \Phi | \hat{O}_V e^{-i\hat{N}_V \theta} | \Phi \rangle,$$

where $P_n$ is the particle-number projection operator,

$$\hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{i(n-\hat{N}_V)\theta}.$$
Outline

1. Introduction

2. Idea: to analyze reaction products in TDHF with PNP

3. An illustrative example: $^{24}\text{O}+^{16}\text{O}$ reactions

4. Summary and Perspective
Outline

1. Introduction

2. Idea: to analyze reaction products in TDHF with PNP

3. An illustrative example: $^{24}\text{O} + ^{16}\text{O}$ reactions

4. Summary and Perspective
Results: Expectation value of the angular momentum operator

One neutron transfer from \( ^{24}\text{O} \) to \( ^{16}\text{O} \); \( ^{24}\text{O} + ^{16}\text{O} \) at \( E_{\text{lab}}=2, 4, 8 \text{ MeV/A}, b \neq 0 \text{ fm} \)

\[
J^V_{z,N,Z} \equiv \left\langle \Phi \right| \hat{J}^V_{z,N} \hat{P}_N^{(n)} \hat{P}_Z^{(p)} \left| \Phi \right\rangle \frac{\left\langle \Phi \right| \hat{P}_N^{(n)} \hat{P}_Z^{(p)} \left| \Phi \right\rangle}{\left\langle \Phi \right| \hat{P}_N^{(n)} \hat{P}_Z^{(p)} \left| \Phi \right\rangle}
\]

\[
\hat{J}_V = \sum_{i=1}^{N} \Theta_V(r_i) [(\hat{r}_i - R_V) \times \hat{p}_i + \hat{s}_i]
\]
Results: **Expected value of the angular momentum operator**

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O}+^{16}\text{O}$ at $E_{\text{lab}}=2, 4, 8 \text{ MeV/A}$, $b\neq 0 \text{ fm}$

\[
J_{V, N, Z} = \frac{\langle \Phi | \hat{J}_{z, V} \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle}{\langle \Phi | \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle}
\]

\[\hat{J}_V = \sum_{i=1}^{N} \Theta_V(\mathbf{r}_i) [ (\hat{r}_i - \mathbf{R}_V) \times \hat{p}_i + \hat{s}_i ]\]

\[d = d(E, b): \text{Distance of closest approach}\]

\[d = \frac{Z_P Z_T e^2}{2E} + \sqrt{\left(\frac{Z_P Z_T e^2}{2E}\right)^2 + b^2}\]

\[\begin{align*}
23\text{O} & (0p, -1n) \\
17\text{O} & (0p, +1n)
\end{align*}\]
**Results:** Expectation value of the angular momentum operator

One neutron transfer from $^{24}$O to $^{16}$O; $^{24}$O+$^{16}$O at $E_{\text{lab}}$=2, 4, 8 MeV/A, $b\neq 0$ fm

\[ J_{z, N, Z}^{V} = \frac{\langle \Phi | \hat{J}_{z, V} \hat{P}^{(n)}_{N} \hat{P}^{(p)}_{Z} | \Phi \rangle}{\langle \Phi | \hat{P}^{(n)}_{N} \hat{P}^{(p)}_{Z} | \Phi \rangle} \]

\[ \hat{J}_{V} = \sum_{i=1}^{N} \Theta_{V}(r_i) [(\hat{r}_i - R_V) \times \hat{p}_i + \hat{s}_i] \]

- **d=d(E,b): Distance of closest approach**
  \[ d = \frac{Z_P Z T e^2}{2E} + \sqrt{\left(\frac{Z_P Z T e^2}{2E}\right)^2 + b^2} \]

K. Sekizawa
Transfer dynamics in the TDHF theory deduced from particle-number projection method

Wed., 12 November, 2014
**Results:** Expectation value of the angular momentum operator

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O};$ $^{24}\text{O}^+^{16}\text{O}$ at $E_{\text{lab}}=2, 4, 8 \text{ MeV/A}, b \neq 0 \text{ fm}$

\[ J_{z,N,Z}^{V} = \frac{\langle \Phi | \hat{J}_{z,V} \hat{P}^{(n)}_{N} \hat{P}^{(p)}_{Z} | \Phi \rangle}{\langle \Phi | \hat{P}^{(n)}_{N} \hat{P}^{(p)}_{Z} | \Phi \rangle} \]

\[ \hat{J}_{V} = \sum_{i=1}^{N} \Theta_{V}(r_{i}) [(\hat{r}_{i} - \mathbf{R}_{V}) \times \hat{p}_{i} + \hat{s}_{i}] \]

\[ d = d(E,b): \text{Distance of closest approach} \]

\[ d = \frac{Z_{P}Z_{Te}^{2}}{2E} + \sqrt{\left(\frac{Z_{P}Z_{Te}^{2}}{2E}\right)^{2} + b^{2}} \]

- $^{23}\text{O} (0p, -1n)$
- $^{17}\text{O} (0p, +1n)$
Results: Expectation value of the angular momentum operator

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O}^+\text{O}$ at $E_{\text{lab}} = 2, 4, 8 \text{ MeV}/A$, $b \neq 0 \text{ fm}$

K. Sekizawa and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

$J_\theta^{V, N, Z} = \langle \Phi | \mathbf{J}_\theta | \Phi \rangle$

$d = d(E, b)$: Distance of closest approach

$d = \frac{Z_P Z_T e^2}{2E}$

PLF

Macroscopic picture of friction

$\mathbf{P}$

$\mathbf{T}$

$\mathbf{L}$

$\mathbf{\beta}$

$\mathbf{\alpha}$

Wed., 12 November, 2014
Results: Expectation value of the angular momentum operator

One neutron transfer from $^{24}$O to $^{16}$O; $^{24}$O+$^{16}$O at $E_{\text{lab}}$=2, 4, 8 MeV/A, $b\neq 0$ fm

$$J_{z,N,Z}^V \equiv \frac{\langle \Phi | \hat{J}_{z,V} \hat{P}_n^N \hat{P}_Z^p | \Phi \rangle}{\langle \Phi | \hat{P}_n^N \hat{P}_Z^p | \Phi \rangle}$$

$$\hat{J}_V = \sum_{i=1}^{N} \Theta_V(\mathbf{r}_i) [(\hat{\mathbf{r}}_i - \mathbf{R}_V) \times \hat{\mathbf{p}}_i + \mathbf{s}_i]$$

$d=d(E,b)$: Distance of closest approach

$$d = \frac{Z_P Z_T e^2}{2E} + \sqrt{\left(\frac{Z_P Z_T e^2}{2E}\right)^2 + b^2}$$

$^{23}$O ($0p, -1n$)

$^{17}$O ($0p, +1n$)

K. Sekizawa
Transfer dynamics in the TDHF theory deduced from particle-number projection method

Wed., 12 November, 2014
Results: Expectation value of the angular momentum operator

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O}+^{16}\text{O}$ at $E_{\text{lab}}=2, 4, 8 \text{ MeV/A, } b \neq 0 \text{ fm}$

\[ J_{z,N,Z}^{V} \equiv \frac{\langle \Phi | \hat{J}_{z,V} \hat{P}_{N}^{(n)} \hat{P}_{Z}^{(p)} | \Phi \rangle}{\langle \Phi | \hat{P}_{N}^{(n)} \hat{P}_{Z}^{(p)} | \Phi \rangle} \]

\[ \hat{J}_{V} = \sum_{i=1}^{N} \Theta_{V} (r_{i}) [\hat{r}_{i} - \hat{R}_{V}] \times \hat{p}_{i} + \hat{s}_{i} \]

\[ d = \frac{Z_{P} Z_{T} e^{2}}{2E} + \sqrt{\left( \frac{Z_{P} Z_{T} e^{2}}{2E} \right)^{2} + b^{2}} \]

- $E_{\text{lab}}=8 \text{ MeV/A}$
- $E_{\text{lab}}=4 \text{ MeV/A}$
- $E_{\text{lab}}=2 \text{ MeV/A}$
Results: Expectation value of the angular momentum operator

One neutron transfer from $^{24}$O to $^{16}$O; $^{24}$O+$^{16}$O at $E_{\text{lab}}$=2, 4, 8 MeV/A, $b\neq 0$ fm

$$J_{z,N,Z}^V = \frac{\langle \Phi | \hat{J}_{z,V} \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle}{\langle \Phi | \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle}$$

$$\hat{J}_V = \sum_{i=1}^{N} \Theta_V(r_i) \left[ (\hat{r}_i - \mathbf{R}_V) \times \hat{p}_i + \hat{s}_i \right]$$

$d=d(E,b)$: Distance of closest approach

$$d = \frac{Z_P Z_T e^2}{2E} + \sqrt{\left(\frac{Z_P Z_T e^2}{2E}\right)^2 + b^2}$$

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
**Results: Expectation value of the angular momentum operator**

One neutron transfer from $^{24}$O to $^{16}$O; $^{24}$O+$^{16}$O at $E_{lab}=2, 4, 8 \text{ MeV/A}, b \neq 0 \text{ fm}$

\[
J_{z, N, Z}^V \equiv \frac{\langle \Phi | J_{z,V} \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle}{\langle \Phi | \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle} \quad \text{and} \quad \hat{J}_V = \sum_{i=1}^{N} \Theta_V(r_i) \left[ (\hat{r}_i - R_V) \times \hat{p}_i + \hat{s}_i \right]
\]

\[
d = d(E, b) : \text{Distance of closest approach}
\]
\[
d = \frac{Z_P Z_T e^2}{2E} + \sqrt{\left( \frac{Z_P Z_T e^2}{2E} \right)^2 + b^2}
\]

**Graphs:**
- $^{23}$O (0p, -1n)
- $^{17}$O (0p, +1n)
Results: Expectation value of the angular momentum operator

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O}+^{16}\text{O}$ at $E_{\text{lab}}=2, 4, 8 \text{ MeV/}A$, $b \neq 0 \text{ fm}$

$$J_{z,N,Z}^{V} = \frac{\langle \Phi | \hat{J}_{z}^{(n)} \hat{P}_{N}^{(p)} \hat{P}_{Z}^{(p)} | \Phi \rangle}{\langle \Phi | \hat{P}_{N}^{(n)} \hat{P}_{Z}^{(p)} | \Phi \rangle} \quad \hat{J}_{V} = \sum_{i=1}^{N} \Theta_{V}(r_{i}) \left[ (\hat{r}_{i} - \hat{R}_{V}) \times \hat{p}_{i} + \hat{s}_{i} \right]$$

$d=d(E,b)$: Distance of closest approach

$$d = \frac{Z_{p}Z_{T}e^{2}}{2E} + \sqrt{\left( \frac{Z_{p}Z_{T}e^{2}}{2E} \right)^{2} + b^{2}}$$

$^{23}\text{O} (0p, -1n)$

$^{17}\text{O} (0p, +1n)$

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
Results: Expectation value of the angular momentum operator

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O} + ^{16}\text{O}$ at $E_{\text{lab}}=2, 4, 8$ MeV/A, $b \neq 0$ fm

- When one neutron is removed from $^{24}\text{O}$, $J_z$ of neutron removed nuclei, $^{23}\text{O}$, is very small irrespective of the value of $E_{\text{lab}}$ and $d$.

- When one neutron is added to $^{16}\text{O}$, $J_z$ of neutron added nuclei, $^{17}\text{O}$, has finite values which increase as the incident energy increases.

---

Graphs showing $J_z$ vs. $d$ for $^{23}\text{O}$ (0p, -1n) and $^{17}\text{O}$ (0p, +1n) for different values of $d$.
Naive interpretation of the results: transferred angular momentum

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

---

**Projectile:** $^{24}\text{O}$

**Target:** $^{16}\text{O}$

---
Naive interpretation of the results: transferred angular momentum

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

Projectile: $^{24}\text{O}$

Target: $^{16}\text{O}$
Naive interpretation of the results: transferred angular momentum

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

\[ \Delta L_z \approx 0 \]

Since \( d > R_P + R_T \), highest occupied neutrons in \( 2s_{1/2} \) orbital would dominate the transfer

\[ \rightarrow \Delta L_z \text{ is expected to be small} \]
Naive interpretation of the results: transferred angular momentum

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

If transferred neutron was in the $2s_{1/2}$ orbital, $\Delta L_z$ would be determined by the relative velocity.

Projectile: $^{24}\text{O}$

Target: $^{16}\text{O}$
Naive interpretation of the results: transferred angular momentum

K. Sekizawa and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

Naive interpretation of the results: transferred angular momentum

Projectile: $^{24}$O

Target: $^{16}$O

If transferred neutron was in the $2s_{1/2}$ orbital, $\Delta L_z$ would be determined by the relative velocity

$r$: distance from the c.m. of $^{16}$O

$m$: nucleon mass

$\nu_{rel}$: relative velocity at the closest distance
Naive interpretation of the results: transferred angular momentum

\[ \Delta L_z \sim r m v_{\text{rel}} \]

- \( r \): distance from the c.m. of \(^{16}\text{O}\)
- \( m \): nucleon mass
- \( v_{\text{rel}} \): relative velocity at the closest distance

Projectile: \(^{24}\text{O}\)

Target: \(^{16}\text{O}\)

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
Results: Expectation value of excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

One neutron transfer from $^{24}$O to $^{16}$O; $^{24}$O+$^{16}$O at $b=0$ fm (head-on collision)

$$\mathcal{E}_{N,Z}^V = \frac{\langle \Phi | \hat{H}_V^{\text{Skyrme}} \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle}{\langle \Phi | \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle} \quad \quad E_{N,Z}^* \equiv \mathcal{E}_{N,Z}^V - E_{N,Z}^{g.s.}$$
Results: Expectation value of excitation energy

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O}+^{16}\text{O}$ at $b=0$ fm (head-on collision)

$$E_{N,Z}^{*} \equiv E_{N,Z} - E_{N,Z}^{g.s.}$$

$$d = d(E,b=0)$$

$$d = \frac{Z_{P}Z_{Te}^{2}}{E}$$

: distance of closest approach

$^{23}\text{O}$ ($0p, -1n$)

$^{17}\text{O}$ ($0p, +1n$)

K. Sekizawa

Transfer dynamics in the TDHF theory deduced from particle-number projection method

Wed., 12 November, 2014

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC
Results: Expectation value of excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O}^+\text{O}$ at $b=0$ fm (head-on collision)

$$E_{N,Z}^* \equiv \mathcal{E}_{N,Z}^V - E_{N,Z}^{g.s.}$$

$$d = d(E, b=0) = \frac{Z_F Z_T e^2}{E} : \text{distance of closest approach}$$

Expectation value of excitation energy

$$\mathcal{E}_{N,Z}^V = \frac{\langle \Phi | \hat{H}_V^{\text{Skyrme}} \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle}{\langle \Phi | \hat{P}_N^{(n)} \hat{P}_Z^{(p)} | \Phi \rangle}$$

$^2^3\text{O} (0p, -1n)$

$^1^7\text{O} (0p, +1n)$
Results: Expectation value of excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O}+^{16}\text{O}$ at $b=0$ fm (head-on collision)

$$\mathcal{E}_{N,Z}^{V} = \frac{\langle \Phi \big| \hat{H}_{V}^{\text{Skyrme}} \hat{\mathbb{P}}_{N}^{(n)} \hat{\mathbb{P}}_{Z}^{(p)} \big| \Phi \rangle}{\langle \Phi \big| \hat{\mathbb{P}}_{N}^{(n)} \hat{\mathbb{P}}_{Z}^{(p)} \big| \Phi \rangle}$$

$$E_{N,Z}^{*} \equiv \mathcal{E}_{N,Z}^{V} - E_{N,Z}^{g.s.}$$

$$d = d(E, b=0) = \frac{Z_{p}Z_{T}e^{2}}{E}$$ : distance of closest approach

Graphs:
- $^{23}\text{O} (0p, -1n)$
- $^{17}\text{O} (0p, +1n)$
**Results: Expectation value of excitation energy**

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

One neutron transfer from $^{24}\text{O}$ to $^{16}\text{O}$; $^{24}\text{O}+^{16}\text{O}$ at $b=0$ fm (*head-on collision*).

- ✔ When one neutron is removed from $^{24}\text{O}$, excitation energy of $^{23}\text{O}$ decreases as $d$ increases.
- ✔ When one neutron is added to $^{16}\text{O}$, excitation energy of $^{17}\text{O}$ increases as $d$ increases.

![Graphs showing energy as function of d for $^{23}\text{O}$ and $^{17}\text{O}$](image)

K. Sekizawa
Transfer dynamics in the TDHF theory deduced from particle-number projection method

Wed., 12 November, 2014
Naive interpretation of the results: average excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

---

![Graph showing $E^*_\text{n,z}$ vs. $d$ for $^{23}\text{O}$ and $^{17}\text{O}$ transitions.]

---

(a) $^{16}\text{O}$

(b) $^{24}\text{O}$

---

K. Sekizawa
Transfer dynamics in the TDHF theory deduced from particle-number projection method

Wed., 12 November, 2014
Naive interpretation of the results: average excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

When $d$ is large, neutrons in $2s_{1/2}$ in $^{24}\text{O}$ would dominate the transfer.

Explanation for $d \geq 10$ fm region:

When $d$ is large, neutrons in $2s_{1/2}$ in $^{24}\text{O}$ would dominate the transfer.
**Naive interpretation of the results:** average excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

When $d$ is large, neutrons in $2s_{1/2}$ in $^{24}\text{O}$ would dominate the transfer.

If energy conserving transfer process occur, $^{17}\text{O}$ will be excited, while $^{23}\text{O}$ will not.
Naive interpretation of the results: average excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

When $d$ is large, neutrons in $2s_{1/2}$ in $^{24}$O would dominate the transfer.

If energy conserving transfer process occur, $^{17}$O will be excited, while $^{23}$O will not.

Explanation for $d \geq 10$ fm region:
Naive interpretation of the results: average excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

Explanation for $d = 9.5$ fm:

When $d$ is small, neutrons in $1d_{5/2}$ in $^{24}O$ would also be transferred.
Naive interpretation of the results: average excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

When $d$ is small, neutrons in $1d_{5/2}$ in $^{24}\text{O}$ would also be transferred.

Explanation for $d = 9.5$ fm:

When $d$ is small, neutrons in $1d_{5/2}$ in $^{24}\text{O}$ would also be transferred. If energy conserving transfer process occur, $^{17}\text{O}$ will not be excited, while $^{23}\text{O}$ will be excited.
Naive interpretation of the results: average excitation energy

K.S. and K. Yabana, arXiv:1409.1083 [nucl-th]; submitted to PRC

When $d$ is small, neutrons in $\text{1d}_{5/2}$ in $^{24}\text{O}$ would also be transferred.

Explaination for $d = 9.5$ fm:

If energy conserving transfer process occur, $^{17}\text{O}$ will not be excited, while $^{23}\text{O}$ will be excited.
Outline

1. Introduction

2. Idea: to analyze reaction products in TDHF with PNP

3. An illustrative example: $^{24}\text{O} + ^{16}\text{O}$ reactions

4. Summary and Perspective
1. Introduction

2. Idea: to analyze reaction products in TDHF with PNP

3. An illustrative example: $^{24}\text{O}+^{16}\text{O}$ reactions

4. Summary and Perspective
Summary

✔ We have studied MNT reactions employing the TDHF theory combined with PNP.

✔ We have developed a theoretical framework to calculate expectation values of operators in the TDHF wave function after collision with the PNP.

✔ To show usefulness of our method, the method is applied to $^{24}\text{O}+^{16}\text{O}$ reaction for two quantities, angular momentum and excitation energy.
**Summary**

- We have studied MNT reactions employing the TDHF theory combined with PNP. ([K.S. and K. Yabana, PRC88(2013)014614, arXiv.1403.2862, 1409.8612, 1409.1083])

- We have developed a theoretical framework to calculate expectation values of operators in the TDHF wave function after collision with the PNP.

- To show usefulness of our method, the method is applied to $^{24}\text{O} + ^{16}\text{O}$ reaction for two quantities, angular momentum and excitation energy.

**Future work**

- Extension to parity and angular momentum projections
Future work: Extension to parity and angular momentum projections

Gamma-particle coincidence measurements for $^{90}\text{Zr}+^{208}\text{Pb}$ at $E_{\text{lab}}=560$ MeV

Future work: Extension to parity and angular momentum projections

Gamma-particle coincidence measurements for $^{90}\text{Zr}+^{208}\text{Pb}$ at $E_{\text{lab}}=560$ MeV

Future work: Extension to parity and angular momentum projections

Gamma-particle coincidence measurements for $^{90}$Zr+$^{208}$Pb at $E_{\text{lab}}=560$ MeV

Future work: Extension to parity and angular momentum projections

Gamma-particle coincidence measurements for $^{90}\text{Zr}+^{208}\text{Pb}$ at $E_{\text{lab}}=560$ MeV

Summary

✔ We have studied MNT reactions employing the TDHF theory combined with PNP.


✔ We have developed a theoretical framework to calculate expectation values of operators in the TDHF wave function after collision with the PNP.

✔ To show usefulness of our method, the method is applied to $^{24}$O+$^{16}$O reaction for two quantities, angular momentum and excitation energy.

Future work

➢ Extension to parity and angular momentum projections
About me:

Kazuyuki SEKIZAWA

Nuclear Theory Group

Graduate School of Pure and Applied Sciences, University of Tsukuba, Japan

Research Fellow of the JSPS (DC2)

E-mail: sekizawa@nucl.ph.tsukuba.ac.jp

URL: http://wwwnucl.ph.tsukuba.ac.jp/~sekizawa/english/